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**111.** **Proposed by F. P. MATZ, D. Sc., Ph. D., Professor of Mathematics and Astronomy, Irving College, Mechanicsburg, Pa.**

By what per cent. of its original dimensions must a linear yard of steel rail, weighing 60 pounds, be increased so that it may weigh 75 pounds?

**Solution by P. H. PHILBRICK, C. E., Lake Charles, La.; SYLVESTER ROBINS, North Branch, N. J., and ELMER SCHUYLER, Annapolis, Md.**

The dimensions of the enlarged rail would be $(\frac{75}{60})^{\frac{1}{3}}=(\frac{10}{8})^{\frac{1}{3}}=\frac{2{\cdot}1544}{2}=$ 1.0772 times those of the original rail, or an increase of 7.72 per cent.

Also solved by *G. B. M. ZERR*, *B. F. SINE*, and *J. F. TRAVIS*.

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## ALGEBRA.

**93.** **Proposed by ELMER SCHUYLER, High Bridge, N. J.**

Given $x^2-yz=1$ ; $y^2-xz=2$ ; $z^2-xy=3$. Find $x$, $y$, and $z$.

**I.** **Solution by C. HORNUNG, A. M., Professor of Mathematics, Heidelberg University, Tiffin, O.**

Subtracting the first equation from the second, the second from the third, and factoring, we get $x+y+z=\dfrac{1}{y-x}$ and $x+y+z=\dfrac{1}{z-y}$, respectively ; whence $y=\dfrac{x+z}{2}$. Substituting this value of $y$ in the equations we get from the first and third $z^2-x^2=\frac{4}{3}$ and from the second $(x-z)^2=8$. From these two equations $x$ and $z$ are easily found to be $\mp\frac{5}{6}\sqrt{2}$ and $\pm\frac{7}{6}\sqrt{2}$, respectively ; whence $y=\pm\frac{1}{6}\sqrt{2}$.

**II.** **Solution by R. E. GAINES, A. M., Professor of Mathematics, Richmond College, Richmond, Va.**

Squaring the first and subtracting the product of the other two, we have

$$x(x^3+y^3+z^3-3xyz)=-5.$$

Similarly, $y(x^3+y^3+z^3-3xyz)=1.$

$$z(x^3+y^3+z^3-3xyz)=7.$$

$$\therefore\ \frac{x}{-5}=\frac{y}{1}=\frac{z}{7}=\pm\sqrt{\left(\frac{x^2-yz}{25-7}\right)}=\pm\sqrt{\tfrac{1}{18}}=\pm\tfrac{1}{6}\sqrt{2}.$$

*i. e.* $x=\mp\frac{5}{6}\sqrt{2}$, $y=\pm\frac{1}{6}\sqrt{2}$, $z=\pm\frac{7}{6}\sqrt{2}$.

[See Charles Smith's *Treatise on Algebra*, page 167].

**III.** **Solution by A. H. BELL, Hillsboro, Ill., and J. SCHEFFER, A. M., Hagerstown, Md.**

$(1)+(2)+(3)=x^2+y^2+z^2-(xy+xz+yz)=6$ . . . . (4).

$(4)^2$ $(x^2+y^2+z^2)^2-(xy+xz+yz)^2-12(xy+xz+yz)=36$. . . . . . (5).

$(1)^2+(2)^2+(3)^2$, $(x^2+y^2+z^2)^2-(xy+xz+yz)^2=14$. . . . . (6).

$(6)-(5)$, $(x+y+z)(xy+xz+yz)=-\frac{11}{6}$. . . . . . (7).

$3\times(7)+(4)$, $(x+y+z)^2=\frac{1}{2}$ or $x+y+z=\sqrt{\frac{1}{2}}=\frac{1}{2}\sqrt{2}$. . . . . . (8).

$2\times(7)+(4)$, $x^2+y^2+z^2+xy+xz+yz=\frac{7}{3}$......(9).

$(1)-(2)-(3)$, $x^2-y^2-z^2+xy+xz-yz=-4$..... (10).

$(9)+(10)$, $x(x+y+z)=-\frac{5}{6}$ by (8) $x=\mp\frac{5}{6}\sqrt{2}$......(11).

$(2)-(1)-(3)+(9)$ etc., $y=\pm\frac{1}{6}\sqrt{2}$......(12).

$(3)-(1)-(2)+(9)$ etc., $z=\pm\frac{7}{6}\sqrt{2}$......(13).

### IV. Solution by G. I. HOPKINS, Manchester, N. H.

Subtract (1) from (2) and (3).

$y^2-x^2+yz-xz=1$......(4).

$z^2-x^2+yz-xy=2$......(5).

Factoring $(y-x)(y+x+z)=1$......(6).

$(z-x)(z+x+y)=2$......(7). Whence $y=\dfrac{x+z}{2}$.

Substituting in (1) and (3) and letting $z=vx$,

$$x^2=\frac{2}{2-v-v^2}=\frac{6}{2v^2-v-1}\text{......(8).}$$

Whence $v=1$ or $-\frac{7}{5}$..... (9).

For $v=1$, leads to indeterminate results.

For $v=-\frac{7}{5}$, $x=\pm\frac{5}{6}\sqrt{2}$, $y=\pm\frac{1}{6}\sqrt{2}$, $z=\pm\frac{7}{6}\sqrt{2}$.

### V. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let $y=ax$, $z=bx$.

$\therefore$ $x^2-abx^2=1$, $a^2x^2-bx^2=2$, $b^2x^2-ax^2=3$.

$\therefore$ $x^2=1/(1-ab)=2/(a^2-b)=3(b^2-a)$.

$\therefore$ $a^2-b=2-2ab$, $b^2-a=3-3ab$.

$\therefore$ $b=(a^2-2)/(1-2a)$.

$\therefore$ $5a^4+a^3-5a-1=0$.

$\therefore$ $(5a^4+1)(a^3-1)=0$.

$\therefore$ $a=-\frac{1}{5}$, $a=1$, $a=-\frac{1}{2}(1\mp\sqrt{-3})$, $b=-\frac{7}{5}$, $b=1$, $b=-\frac{1}{2}(1\pm\sqrt{-3})$.

The second and third values of $a$ and $b$ give $x=y=z=\infty$ and are therefore not admissible.

$$a=-\tfrac{1}{5},\ b=-\tfrac{7}{5},\ \text{give } x=\pm\frac{5}{3\sqrt{2}},\quad y=\mp\frac{1}{3\sqrt{2}},\ z=\mp\frac{7}{3\sqrt{2}}.$$

Also solved by ***P. S. BERG, J. M. BOORMAN, W. H. DRANE, ALOIS F. KOVARIK, CHARLES E. MEYERS, H. N. HERRICK, NELSON L. RORAY, E. D. SCALES***, and the ***PROPOSER***.